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The First Mathematics Task

Our student teachers arrive *en masse* – all subjects in a central common room – and are led by mathematics tutors to the teaching room. As they enter the room a tutor sticks a post-it on each forehead and instructs them that they have to find out what number they have stuck to their foreheads by asking each other questions to which the answer is 'yes' or 'no'. They form themselves naturally into groups as they do this task, and begin to ask questions. The secret is that each of them has the number '1' on their foreheads, but it is disguised in various mathematical forms such as

$:3^0$, $\sin^2 x + \cos^2 x$, $0.\dot{9}$, $e^{i\pi}$ and so on.

This becomes clear before long and people by that time are generally chatting, either about other expressions which might equal 1, or about themselves. It is common to see people walking around 'collecting' foreheads.

Then they sit at tables in fours and write their names on sticky labels to wear for the rest of the day. We ask them to introduce themselves to each other and talk about how they felt about the task.

We have deliberately used some challenging expressions from A-level and beyond in this task, as well as a few easier ones, because we hope that students will recognise the value of being immersed in mathematical challenges from the start. We ask 'how was that?' and there are always a few who comment that they had a hard time remembering the relevant mathematics, and this has made them aware they need to work on some topic or other. This articulation has a purpose – it shows that not everyone in the room is a superb mathematician; that struggle is OK; that choosing to take responsibility for working on mathematics is an appropriate response; that getting stuck in mathematics does not need to be embarrassing; that discussion in groups is useful; that a 'fun' element in mathematics does not need to be trivial or easy; and so on.

More discussion tasks follow and we also issue a list of all the '1' questions with comments about why some of them have been chosen and asking them to note how powerful and intriguing it can be to think about mathematics when you already know the answer**.**

Half the number of triangles which can be constructed with sides of 7cm and 10 cm and a non-included angle of 33 degrees?	how can you get your pupils to think of *all* possibilities in any situation?
The limit of the sequence: 1, 1, 1, 1, ……..	what range of examples of sequences do learners need to fully understand that limits are not always to do with getting very close to something?
12321 ÷ 111 ÷ 111	what is the value of this kind of number pattern? Can this be written as 12321/111/111 and be correct?
$2359 – (2 \times 10^3 + 3 \times 10^2 + 5 \times 10 + 8)$	knowing that any integer can be written as the sum of multiples of powers of ten can be a good introduction to standard form
$\sin^2 (\pi/3) + \cos^2 (\pi/3)$	how can you help learners recognise this identity without having to look it up on a formula sheet?
$\sin^2 47 + \cos^2 47$	when is it NOT alright to express angles in degrees for using trigonometric functions?
$\sec^2 (2\pi/5) - \tan^2 (2\pi/5)$	how can you help learners recognise trig identities in non-standard form?
$\lim_{n \to \infty} \sqrt{}\sqrt{}\sqrt{}\sqrt{}\ldots\ldots_{n\ times}\sqrt{9}$	exploration with a calculator can be a source of mathematical insight
$\lim_{n \to \infty} \sqrt{}\sqrt{}\sqrt{}\sqrt{}\ldots\ldots_{n\ times}\sqrt{(1/9)}$	how might you explain that this limit is the same as the one above?

value of x which makes all of these zero: $x^2 - (a + 1)x + a$	how can exploring properties of particular types of function help learners understand the overall type?
$- i^2$	worth recognising ?
i^4	
$\frac{1}{2}(1 - i - i^2 - i^3 - i^4 - i^5 - i^6 - i^7)$	would this help learners develop fluency with i?
Remainders when 5,13,17,25 are each divided by 4	why give several examples? Because it is intriguing for learners to see that numbers which are congruent modulo n form a set of n classes which include all integers less than n.
Order of rotational symmetry of a heart-shape	how can you find out whether it is the words or the sense of symmetry which lead to wrong answers?
Number of axes of symmetry of an isosceles trapezium	would learners recognise this if it were not sitting comfortably on a base parallel to the bottom of the page?
$\frac{129}{301} \div \frac{3}{7}$	would you accept this answer: $\frac{129 \div 3}{301 \div 7} = \frac{43}{43} = 1$?
(8 – 7 + 9 – 8)/2	when we use *BODMAS,* we are telling pupils to add before subtracting, is that helpful?
$0.\dot{9}$	how do we convince learners that this is exactly 1, not approximately 1?
tan 45	is it important to commit standard results, like this one, to memory, when there are always calculators nearby? Devise a task in which *knowing* this is crucial
tan (π/4)	what understanding of pi would a learner need to have for this to make sense?
- sin(270)	how does the mnemonic '*sohcahtoa*' help you understand what this is?
$48/(2^4 \times 3)$	when might it be helpful to 'see' composite numbers as multiplicative structures?
Value of x when 5x + 2 = 7	why would anyone ever be motivated to learn a 'method' to solve a linear equation like this one?
Value of x when 22 = 2 (x + 10)	note that in this one the equals sign cannot be interpreted to mean 'and the answer is…'
Number of sides in a polygon divided by the number of angles	what experiences would a learner need to believe this generalisation?
Number of different triangles which can be constructed with sides of 4, 5, and 6 cm.	what does it mean for triangles to be the same or different? And what did *you* think of first when you saw the 4 and 5?
The gradient of x – y = 7?	if learners only meet straight line graphs in the usual way, y = mx + c, they may not recognise them in other forms. When would this form be more useful?
The number of even prime numbers	the sort of 'catch' question teachers use to check their students know what the words mean